Lecture 12

Hypothesis Testing
Outline

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- 9-7 Confidence Intervals and Hypothesis Testing
9-1 Introduction

- Hypothesis testing is a decision-making process for evaluating claims about a population.
- We must define the population under study,
- state the particular hypotheses that will be investigated,
- give the significance level,
- select a sample from the population,
- collect the data,
- perform the calculations required for the statistical test,
- reach a conclusion.
9-2 Steps in Hypothesis Testing

- A **Statistical hypothesis** is a conjecture about a population parameter. This conjecture may or may not be true.
- The **null hypothesis**, symbolized by $H_0$, is a statistical hypothesis that states that there is no difference between a parameter and a specific value or that there is no difference between two parameters.
The alternative hypothesis, symbolized by $H_1$, is a statistical hypothesis that states a specific difference between a parameter and a specific value or states that there is a difference between two parameters.
A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication.
Steps in Hypothesis Testing - Example

- What are the hypotheses to test whether the pulse rate will be different from the mean pulse rate of 82 beats per minute?
  - $H_0: \mu = 82 \quad H_1: \mu \neq 82$
  - This is a two-tailed test.
9-2 Steps in Hypothesis Testing - Example

- A chemist invents an additive to increase the life of an automobile battery. If the mean lifetime of the battery is 36 months, then his hypotheses are

- \( H_0: \mu \leq 36 \quad H_1: \mu > 36 \)
- This is a right-tailed test.
9-2 Steps in Hypothesis Testing - Example

- A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is $78, her hypotheses about heating costs will be
  - $H_0$: $\mu \geq 78$
  - $H_0$: $\mu < 78$
- This is a left-tailed test.
9-2 Steps in Hypothesis Testing

- A **statistical test** uses the data obtained from a sample to make a decision about whether or not the null hypothesis should be rejected.
9-2 Steps in Hypothesis Testing

- The numerical value obtained from a statistical test is called the **test value**.
- In the hypothesis-testing situation, there are four possible outcomes.
9-2 Steps in Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.
### 9-2 Steps in Hypothesis Testing

<table>
<thead>
<tr>
<th></th>
<th>$H_0$ True</th>
<th>$H_0$ False</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$</td>
<td>Error Type I</td>
<td>Correct decision</td>
</tr>
<tr>
<td>Do not reject $H_0$</td>
<td>Correct decision</td>
<td>Error Type II</td>
</tr>
</tbody>
</table>
9-2 Steps in Hypothesis Testing

- A type I error occurs if one rejects the null hypothesis when it is true.
- A type II error occurs if one does not reject the null hypothesis when it is false.
The level of significance is the maximum probability of committing a type I error. This probability is symbolized by $\alpha$ (Greek letter alpha). That is, $P(\text{type I error}) = \alpha$.

$P(\text{type II error}) = \beta$ (Greek letter beta).
9-2 Steps in Hypothesis Testing

- Typical significance levels are: 0.10, 0.05, and 0.01.
- For example, when $\alpha = 0.10$, there is a 10% chance of rejecting a true null hypothesis.
9-2 Steps in Hypothesis Testing

- The **critical value(s)** separates the critical region from the noncritical region.
- The symbol for critical value is C.V.
9-2 Steps in Hypothesis Testing

- The critical or rejection region is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.
9-2 Steps in Hypothesis Testing

The noncritical or nonrejection region is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.
9-2 Steps in Hypothesis Testing

- A one-tailed test (right or left) indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean.
9-2 Finding the Critical Value for \( \alpha = 0.01 \) (Right-Tailed Test)

\[ z = 2.33 \]

Critical region

Noncritical region

\( 0 \)

\( \alpha = 0.01 \)

9-2 Finding the Critical Value for \( \alpha = 0.01 \) (Left-Tailed Test)

- For a left-tailed test when \( \alpha = 0.01 \), the critical value will be \(-2.33\) and the critical region will be to the left of \(-2.33\).
9-2 Finding the Critical Value for \( \alpha = 0.01 \) (Two-Tailed Test)

- In a two-tailed test, the null hypothesis should be rejected when the test value is in either of the two critical regions.
Finding the Critical Value for $\alpha = 0.01$ (Two-Tailed Test)

$\alpha / 2 = 0.005$

Critical region

Noncritical region

$z = -2.58$  $0$  $z = 2.58$
3 Large Sample Mean Test

(z test)

- The **z test** is a statistical test for the mean of a population. It can be used when \( n \geq 30 \), or when the population is normally distributed and \( \sigma \) is known.
- The formula for the **z test** is given on the next slide.
9-3 Large Sample Mean Test

\[ z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \]

where
\( \bar{X} = \) sample mean
\( \mu = \) hypothesized population mean
\( \sigma = \) population deviation
\( n = \) sample size
Five steps for hypothesis-testing:

- State the hypotheses and identify the claim.
- Find the critical value(s).
- Compute the test value.
- Make the decision to reject or not reject the null hypothesis.
- Summarize the results.
A researcher reports that the average salary of assistant professors is more than $42,000. A sample of 30 assistant professors has a mean salary of $43,260. At $\alpha = 0.05$, test the claim that assistant professors earn more than $42,000 a year. The standard deviation of the population is $5230$. 
9-3 Large Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
  - $H_0: \mu \leq 42,000$  \quad  $H_1: \mu > 42,000$ (claim)

- **Step 2:** Find the critical value. Since $\alpha = 0.05$ and the test is a right-tailed test, the critical value is $z = +1.65$.

- **Step 3:** Compute the test value.
Step 3: \[ z = \frac{43,260 - 42,000}{\sqrt{5230/30}} \]
\[ = 1.32. \]

Step 4: Make the decision. Since the test value, +1.32, is less than the critical value, +1.65, and not in the critical region, the decision is “Do not reject the null hypothesis.”
Step 5: Summarize the results. There is not enough evidence to support the claim that assistant professors earn more on average than $42,000 a year.

See the next slide for the figure.
9-3 Large Sample Mean Test - Example

Example

\[ \alpha = 0.05 \]

Reject

\[ \alpha = 0.05 \]
A national magazine claims that the average college student watches less television than the general public. The national average is 29.4 hours per week, with a standard deviation of 2 hours. A sample of 30 college students has a mean of 27 hours. Is there enough evidence to support the claim at $\alpha = 0.01$?
Step 1: State the hypotheses and identify the claim.

\[ H_0: \mu \geq 29.4 \quad H_1: \mu < 29.4 \quad \text{(claim)} \]

Step 2: Find the critical value. Since \( \alpha = 0.01 \) and the test is a left-tailed test, the critical value is \( z = -2.33 \).

Step 3: Compute the test value.
9-3 Large Sample Mean Test - Example

- **Step 3:** \( z = \frac{[27 - 29.4]}{\sqrt{\frac{2}{30}}} = -6.57 \).
- **Step 4:** Make the decision. Since the test value, \(-6.57\), falls in the critical region, the decision is to reject the null hypothesis.
9-3  Large Sample Mean Test - Example

- **Step 5:** Summarize the results. There is enough evidence to support the claim that college students watch less television than the general public.
- See the next slide for the figure.
9-3 Large Sample Mean Test - Example

-6.57

-2.33

Reject
9-3 Large Sample Mean Test - Example

- The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a large hospital, a researcher selected a random sample of 35 stroke victims and found that the average cost of their rehabilitation is $25,226.
The standard deviation of the population is $3,251. At $\alpha = 0.01$, can it be concluded that the average cost at a large hospital is different from $24,672$?

**Step 1:** State the hypotheses and identify the claim.

- $H_0: \mu = 24,672$  
- $H_1: \mu \neq 24,672$ (claim)
9-3 Large Sample Mean Test - Example

- **Step 2:** Find the critical values. Since $\alpha = 0.01$ and the test is a two-tailed test, the critical values are $z = -2.58$ and $+2.58$.
- **Step 3:** Compute the test value.
- **Step 3:** $z = \frac{[25,226 - 24,672]}{[3,251/\sqrt{35}]} = 1.01$. 
9-3 Large Sample Mean Test - Example

- **Step 4:** Make the decision. Do not reject the null hypothesis, since the test value falls in the noncritical region.

- **Step 5:** Summarize the results. There is not enough evidence to support the claim that the average cost of rehabilitation at the large hospital is different from $24,672.
9-3 Large Sample Mean Test - Example
9-3 *P*-Values

- Besides listing an \( \alpha \) value, many computer statistical packages give a *P*-value for hypothesis tests.
- The *P*-value is the actual probability of getting the sample mean value or a more extreme sample mean value in the direction of the alternative hypothesis (> or <) if the null hypothesis is true.
9-3 *P*-Values

- The *P*-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample mean or a more extreme sample mean occurring if the null hypothesis is true.
9-3 \( P \)-Values – decision rule

- If \( P \)-value \( \leq \alpha \), reject the null hypothesis.
- If \( P \)-value \( > \alpha \), do not reject the null hypothesis.
A researcher wishes to test the claim that the average age of lifeguards in Ocean City is greater than 24 years. She selects a sample of 36 guards and finds the mean of the sample to be 24.7 years, with a standard deviation of 2 years. Is there evidence to support the claim at $\alpha = 0.05$? Find the $P$-value.
9-3 *P*-Values - Example

- **Step 1:** State the hypotheses and identify the claim.
  - $H_0: \mu \leq 24 \quad H_1: \mu > 24$ (claim)
- **Step 2:** Compute the test value.
  
  \[
  z = \frac{24.7 - 24}{\frac{2}{\sqrt{36}}} = 2.10
  \]
Step 3: Using Table E in Appendix C, find the corresponding area under the normal distribution for $z = 2.10$. It is 0.4821

Step 4: Subtract this value for the area from 0.5000 to find the area in the right tail.

$$0.5000 - 0.4821 = 0.0179$$

Hence the $P$-value is 0.0179.
Step 5: Make the decision. Since the $P$-value is less than 0.05, the decision is to reject the null hypothesis.

Step 6: Summarize the results. There is enough evidence to support the claim that the average age of lifeguards in Ocean City is greater than 24 years.
A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the sample is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the $P$-value method.
9-3  $P$-Values - Example

- **Step 1:** State the hypotheses and identify the claim.
  - $H_0: \mu = 8$ (claim)  \hspace{0.5cm} H_1: \mu \neq 8

- **Step 2:** Compute the test value.
  \[
  z = \frac{8.2 - 8}{0.6/\sqrt{32}} = 1.89
  \]
9-3 $P$-Values - Example

- **Step 3:** Using table E, find the corresponding area for $z = 1.89$. It is 0.4706.
- **Step 4:** Subtract the value from 0.5000. 
  $0.5000 - 0.4706 = 0.0294$
Step 5: Make the decision: Since this test is two-tailed, the value 0.0294 must be doubled; $2(0.0294) = 0.0588$. Hence, the decision is not to reject the null hypothesis, since the $P$-value is greater than 0.05.

Step 6: Summarize the results. There is not enough evidence to reject the claim that the average wind speed is 8 miles per hour.
9-4 Small Sample Mean Test

\( t \) test

- When the population standard deviation is unknown and \( n < 30 \), the \( z \) test is inappropriate for testing hypotheses involving means.
- The \( t \) test is used in this case.
- Properties for the \( t \) distribution are given in Chapter 8.
9-4 Small Sample Mean Test - Formula for $t$ test

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

where

$\bar{X} = \text{sample mean}$

$\mu = \text{hypothesized population mean}$

$s = \text{sample standard deviation}$

$n = \text{sample size}$

degrees of freedom $= n - 1$
A job placement director claims that the average starting salary for nurses is $24,000. A sample of 10 nurses has a mean of $23,450 and a standard deviation of $400. Is there enough evidence to reject the director’s claim at \( \alpha = 0.05 \)?
9-4 Small Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
  - $H_0: \mu = $24,000 (claim)  
  - $H_1: \mu \neq $24,000 

- **Step 2:** Find the critical value. Since $\alpha = 0.05$ and the test is a two-tailed test, the critical values are $t = -2.262$ and $+2.262$ with d.f. = 9.
9-4 Small Sample Mean Test - Example

- **Step 3:** Compute the test value.
  \[ t = \frac{23,450 - 24,000}{400/\sqrt{10}} = -4.35. \]

- **Step 4:** Reject the null hypothesis, since 
  \[ -4.35 < -2.262. \]

- **Step 5:** There is enough evidence to 
  reject the claim that the starting salary of nurses is $24,000.
9-3 Small Sample Mean Test - Example

-4.35
-2.262
2.262
9-7 Confidence Intervals and Hypothesis Testing

- When the null hypothesis is rejected in a hypothesis-testing, the confidence interval for the mean using the same level of significance will not contain the hypothesized mean.
- Likewise, when the null hypothesis is not rejected, the confidence interval computed using the same level of significance will contain the hypothesized mean.
Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated, at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.
9-7 Confidence Intervals and Hypothesis Testing - Example

- $H_0: \mu = 5 \quad H_1: \mu \neq 5$ (claim)
- The critical values are $+1.96$ and $-1.96$
- The test value is $z = \frac{4.6 - 5.00}{0.7/\sqrt{50}} = -4.04$
- Since $-4.04 < -1.96$, the null hypothesis is rejected.
- There is enough evidence to support the claim that the bags do not weigh 5 pounds.
9-7 Confidence Intervals and Hypothesis Testing - Example

- The 95% confidence interval for the mean is given by
  \[
  \bar{X} - z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}
  \]
  
  \[
  4.6 - (1.96) \left( \frac{0.7}{\sqrt{50}} \right) < \mu < 4.6 + (1.96) \left( \frac{0.7}{\sqrt{50}} \right)
  \]
  
  \[
  4.406 < \mu < 4.794
  \]

- Notice that the 95% confidence interval of \( \mu \) does not contain the hypothesized value \( \mu = 5 \). Hence, there is agreement between the hypothesis test and the confidence interval.
Using the $z$ or $t$ test

Is $\sigma$ known?

- Yes: $z = \frac{X - \mu}{\sigma / \sqrt{n}}$
- No:
  - Is $n \geq 30$?
    - Yes: $z = \frac{X - \mu}{s / \sqrt{n}}$
    - No: $t = \frac{X - \mu}{s / \sqrt{n}}$

See Note

Note: With d.f. = $n - 1$, and the population must be approximately normally distributed.